FRACTIONS

Summary:

1. A fraction is a number in the form $\frac{a}{b}$.

2. In a fraction the top number is called the numerator and the bottom number is called the denominator

3. In a proper fraction the numerator is less than the denominator. Thus $\frac{3}{4}$ *and* $\frac{7}{9}$ *are both proper fractions.*

4. In an improper fraction the numerator is greater than the denominator. Thus $\frac{5}{3}$ *and* $\frac{8}{5}$ *are both improper fractions.*

5. In a mixed number a whole number is followed by a proper fraction. Thus $1\frac{3}{4}$ *and* $2\frac{1}{5}$ *are both mixed numbers. A mixed number can be converted into an improper fraction and vice versa*

6. Equivalent fractions have the same value. In an equivalent fraction both the numerator and denominator are multiplied ***or*** *divided by the same number.*

Thus $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$ *and* $\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$

7. A fraction is in simplest form (lowest terms) when the top and bottom cannot be any smaller.

8. In comparing fractions the one with a larger percentage is the largest

EXAMPLES:

1. *Convert* $3\frac{2}{5}$ *into an improper fraction*

Solution

$$3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$$

2. *Express* $\frac{11}{4}$ *as a mixed number*

Solution

$\frac{11}{4} = 2\frac{3}{4}$ *(since* $11 \div 4 = 2$ *remainder* 3*)*

3. *Reduce* $\frac{21}{35}$ *to its simplest form*

Solution

$$\frac{21}{35} = \frac{21 \div 7}{35 \div 7} = \frac{3}{5}$$

4. *Arrange the fractions* $\frac{2}{3}$, $\frac{3}{7}$, $\frac{3}{4}$ *and* $\frac{2}{5}$ *in ascending order of magnitude*

5. *Arrange the fractions* $\frac{5}{6}$, $\frac{4}{9}$, $\frac{7}{8}$ *and* $\frac{1}{2}$ *in descending order of magnitude*

OPERATIONS WITH FRACTIONS

Summary:

*(i)For fractions with plus **(+)** and minus **(–)** signs only, find the **LCM** and workout*

*(ii) For fractions with combined operations, the **BODMAS** rule must be observed.*

EXAMPLES:

***1.** Without using a calculator, express the following as a single fraction:*

***(a)** $\frac{2}{5} + \frac{3}{7}$ **(b)** $1\frac{2}{3} + 2\frac{1}{4}$ **(c)** $\frac{5}{6} - \frac{3}{4}$ **(d)** $6\frac{3}{4} - 4\frac{1}{3}$*

***(e)** $\frac{1}{2} - \frac{3}{4} + \frac{5}{8} - \frac{7}{16} + \frac{19}{32}$ **(f)** $\frac{2}{3}$ *of* $\frac{4}{5}$ **(g)** $\frac{3}{8} \times \frac{4}{9}$*

***(h)** $\frac{2}{3} \times \frac{5}{7} \times \frac{21}{32}$ **(i)** $1\frac{7}{8} \times 3\frac{1}{5}$ **(j)** $\frac{5}{9} \div \frac{2}{3}$ **(k)** $\frac{3}{8} \div 2\frac{1}{4}$*

***(l)** $1\frac{1}{2} \div 2\frac{1}{4}$ **(m)** $21 \div \frac{7}{9}$ **(n)** $1\frac{2}{7} \div 6$*

***2.** Without using a calculator, simplify the following fractions:*

***(i)** $\left(2\frac{1}{2} - 1\frac{1}{3}\right) \div 1\frac{5}{9}$ **(ii)** $\frac{2}{5}$ *of* $\left(\frac{2}{3} - \frac{1}{4}\right) + \frac{1}{2}$*

***(iii)** $\dfrac{\frac{4}{5} - \frac{1}{3} \div \frac{1}{2}}{\frac{2}{5}\ \text{of}\ \frac{3}{4} + 3\frac{1}{2}}$ **(iv)** $\dfrac{\frac{2}{3}\ \text{of}\ \left(1\frac{3}{7} - \frac{5}{8}\right)}{\frac{3}{4} + 1\frac{5}{7} \div \frac{4}{7}\ \text{of}\ 2\frac{1}{3}}$*

(v) $2\frac{1}{2} \div \frac{4\frac{1}{3} - 2\frac{1}{4}}{4\frac{1}{6}}$ **(vi)**

$$\frac{\frac{3}{5} \text{ of } 60 - 2\frac{2}{3} \times 1\frac{1}{2}}{5\frac{5}{8} \times 1\frac{7}{9} - \frac{5}{4} \text{ of } 4\frac{4}{5} + 2\frac{4}{5} \div \frac{7}{10}}$$

(vii) $8 \text{ of } \left\{(\frac{3}{4} + 1\frac{5}{7} \div \frac{4}{7} \text{ of } 2\frac{1}{3}) \div (1\frac{3}{7} - \frac{5}{8}) \text{ of } \frac{2}{3}\right\}$

3. *Express the following as a single fraction:*

(i) $\frac{x}{3} + \frac{x}{4} + \frac{x}{5}$ **(ii)** $\frac{x-2}{4} + \frac{2}{5}$

(iii) $\frac{3x-5}{10} + \frac{2x-3}{15}$ **(iv)** $\frac{x-5}{3} - \frac{x-2}{4}$

(v) $3(x+2) - \frac{4x-5}{4}$ **(vi)** $\frac{2x-5}{5} - \frac{3x+2}{4} + \frac{7x+15}{10}$

FRACTION WORD PROBLEMS

1. *A man gave half of his wealth to his wife, one-fifth to each of his two sons and the rest to his daughter. Find:*

(i) *the fraction given to the daughter*

(ii) *his wealth, if each son received* ***Shs 16,000***

Soln:

(i) *Required fraction* $= 1 - \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{5}\right) = \frac{1}{10}$

(ii) *If* ***W*** *is the man's wealth,*

$$\Rightarrow \quad \frac{1}{5}W = 16{,}000$$

$$\therefore \quad W = 80{,}000$$

2. *A man gave* $\frac{3}{5}$ *of his money to his wife and* $\frac{1}{4}$ *of the remainder to his son. If he was left with* **Shs 90,000,** *find how much did his son receive*

Soln:

Fraction given to wife and son = $\frac{3}{5} + \frac{1}{4}\left(\frac{2}{5}\right) = \frac{7}{10}$

$\Rightarrow$ *Fraction left =* $1 - \frac{7}{10} = \frac{3}{10}$

If **M** *is the man's money,*

$$\Rightarrow \quad \frac{3}{10}M = 90{,}000$$

$$M = 300{,}000$$

$$\therefore \quad \text{Amount given to son} = \frac{1}{4}\left(\frac{2}{5}\right) \times 300{,}000 = \text{Shs } 30{,}000$$

3. *A man spent* $\frac{1}{4}$ *of his salary on fees and* $\frac{2}{5}$ *of the remainder on rent. If he was left with* **Shs 270,000,** *find:*

(i) *the man's salary*

(ii) *how much did he spend on rent*

Soln:

(i) *Fraction on fees and rent =* $\frac{1}{4} + \frac{2}{5}\left(\frac{3}{4}\right) = \frac{11}{20}$

$\Rightarrow$ *Fraction left =* $1 - \frac{11}{20} = \frac{9}{20}$

*If **M** is the man's salary,*

$$\Rightarrow \quad \frac{9}{20} M = 270{,}000$$
$$M = 600{,}000$$

(ii) *Expenditure on rent* $= \frac{2}{5}\left(\frac{3}{4}\right) \times 600{,}000 = \text{Shs } 180{,}000$

4. *In a school,* $\frac{2}{3}$ *of the students are boys.* $\frac{1}{4}$ *of the boys and* $\frac{2}{5}$ *of the girls are left–handed. Find the:*

(i) *fraction of left–handed students in the school*

(ii) *number of students in the school if **420** are left–handed*

Soln:

(i) *Required fraction =* $\frac{2}{3}\left(\frac{1}{4}\right) + \frac{1}{3}\left(\frac{2}{5}\right) = \frac{3}{10}$

(ii) *If **N** is the required number,*

$$\Rightarrow \quad \frac{3}{10} N = 420$$
$$\therefore \quad N = 1{,}400$$

EER:

1. *Simplify without using a calculator:* $\frac{3}{4} + 1\frac{5}{7} \div \frac{4}{7}$ *of* $2\frac{1}{3}$

2. *Express* $\dfrac{2\frac{2}{5} + 1\frac{7}{8}}{\frac{3}{4} - \frac{1}{3}}$ *as a single fraction*

3. *Simplify without using a calculator:* $\dfrac{\frac{4}{11} \text{ of } \left(\frac{3}{4} - \frac{1}{20}\right)}{\left(2 + \frac{1}{3}\right) \div \left(1 + \frac{2}{3}\right)}$

4. *Simplify without using a calculator:* $\dfrac{\frac{1}{2} \text{ of } 3\frac{3}{2} + \frac{3}{2}\left(\frac{5}{2} - \frac{2}{3}\right)}{\frac{3}{4} \text{ of } 2\frac{1}{2} \div \frac{1}{4}}$

5. *Simplify without using a calculator:* $\dfrac{\frac{2}{3}\left(1 - \frac{2}{5} + \frac{1}{4}\right)}{\frac{7}{50} + \frac{1}{3} \text{ of } \frac{3}{5}}$

8. *The numerator of a fraction is 3 less than its denominator. If the numerator is tripled and the denominator is increased by* **7,** *the resulting fraction is* $\frac{3}{2}$ *. Find the original fraction*

6. *Simplify without using a calculator:* $\dfrac{3\frac{2}{3} \text{ of } 21 + 3\frac{3}{4} \div \frac{3}{8} - 4\frac{1}{2} \times 3\frac{1}{3}}{5\frac{5}{8} \times 1\frac{7}{9} - \frac{5}{4} \text{ of } 4\frac{4}{5} + 2\frac{4}{5} \div \frac{7}{10}}$

7. *A man spent* $\frac{3}{5}$ *of his leave in his shop and* $\frac{3}{20}$ *on visits. If he was left with* **15** *more days to spend,*

(i) *find the length of his leave*

(ii) *if after spending* $\frac{3}{5}$ *of his leave in his shop, he spent* $\frac{3}{20}$ *of the remainder on visits and realized that he was left with* **17** *more days to spend, find how many days had he already spent*

8. *Express* $1 - \frac{2x}{5} + \frac{x}{8}$ *as a single fraction*

9. Given that $a * b = \frac{ab}{a+1}$, find the value of $\frac{1}{2} * \frac{1}{3}$

10. Express $\frac{x+3}{3} - \frac{x-7}{4} + \frac{x-5}{2}$ as a single fraction

11. Given the fractions $\frac{3}{5}$, $\frac{3}{4}$, $\frac{7}{10}$ and $\frac{13}{20}$, find the difference between the largest and the smallest fractions

12. Given that $a * b = \frac{a}{b} + \frac{b}{a}$, find the value of $\frac{1}{2} * \frac{2}{3}$

13. Express $\frac{x-3}{3} - \frac{x-7}{4}$ as a single fraction

14. **Shs 12,000** is shared among **P, Q** and **R. P** takes one–fifth of it, **Q** takes one–sixth of the remainder and **R** takes what is left. Find:

(i) the fraction taken by **R**

(ii) how much does each receive

15. Simplify without using a calculator: $\frac{1\frac{1}{2} - \left(8\frac{1}{3} \div 2\frac{1}{2}\right)}{1\frac{1}{5} \text{ of } \left(1\frac{1}{4} + 1\frac{2}{3}\right)}$

16. Express $\frac{2x-5}{5} - \frac{3x+2}{4} + \frac{7x+15}{10}$ as a single fraction

17. Express $\frac{3x-7}{3} - \frac{2x-5}{2}$ as a single fraction

18. Express $4x - \left(\frac{3x+y}{8} - \frac{3x+y}{4}\right)$ as a single fraction

19. Express $\frac{3\frac{1}{4} + \frac{3}{5} \div \frac{5}{17} \text{ of } 3\frac{2}{5}}{1\frac{7}{10}}$ as a single fraction

20. *Simplify without using a calculator:* $\dfrac{\frac{3}{8} - \frac{1}{5}}{\frac{7}{10} - \frac{2}{3}}$

APPLICATION OF FRACTIONS

EXAMPLES:

1. *Bob takes* ***10*** *days to dig a garden. Ben takes* ***15*** *days to dig the same garden. Find how long it will take them to dig the same garden if they work together.*

Soln:

If T = required time

$\Rightarrow \frac{1}{T} = \frac{1}{10} + \frac{1}{15}$

$\therefore$ ***T = 6*** *days*

2. *A basin can be filled separately by taps* ***P*** *and* ***Q*** *in* ***6*** *minutes and* ***12*** *minutes respectively. Find how long the two taps will take to fill the same basin if they are opened at once when the basin is empty.*

Soln:

If T = required time

$\Rightarrow \frac{1}{T} = \frac{1}{6} + \frac{1}{12}$

$\therefore$ ***T = 4*** *minutes*

3. *Tap* ***P*** *can fill a tank in* ***6*** *minutes, while tap* ***Q*** *can empty it in* ***15*** *minutes. Find how long it will take to fill this tank if the two taps are opened at once when the tank is empty.*

Soln:

If T = required time

$$\Rightarrow \frac{1}{T} = \frac{1}{6} - \frac{1}{15}$$

$\therefore$ ***T = 10*** *minutes*

4. *A tank can be filled separately by taps* ***P*** *and* ***Q*** *in* ***6*** *minutes and* ***12*** *minutes respectively. Tap* ***R*** *can empty the same tank in* ***5*** *minutes. Find how long it will take to fill this tank if the taps are opened at once when the tank is empty.*

Soln:

If T = required time

$$\Rightarrow \frac{1}{T} = \frac{1}{6} + \frac{1}{12} - \frac{1}{5}$$

$\therefore$ ***T = 20*** *minutes*

5. *Tom and Bob take* ***12*** *days working together to clear a garden. It takes* ***30*** *days for Tom alone to clear the same garden. Find how long it takes Bob alone to clear this garden.*

Soln:

If T = required time

$$\Rightarrow \frac{1}{12} = \frac{1}{30} + \frac{1}{T}$$

$\therefore$ ***T = 20*** *days*

EER:

1. *A garden can be cleared separately by Bob and Ben in* ***30*** *days and* ***20*** *days respectively. Find how long it will take them to clear the same garden if they work together.*

2. *A basin can be filled separately by taps* ***P*** *and* ***Q*** *in* ***30*** *minutes and* ***45*** *minutes respectively. Find how long it will take to fill this tank if the taps are opened at once when the tank is empty.*

3. *A tank can be filled separately by taps* ***P*** *and* ***Q*** *in* ***12*** *minutes and* ***15*** *minutes respectively. Tap* ***R*** *can empty the same tank in* ***10*** *minutes. Find how long it will take to fill this tank if the taps are opened at once when the tank is empty.*

DECIMALS NUMBERS

Summary:

1. *A decimal number is a number with a decimal point. Thus* ***1·75*** *is a decimal number*

2. *To convert a fraction to decimal, divide the numerator by the denominator. Thus* $\frac{1}{2} = 1 \div 2 = 0 \cdot 5$ *and* $\frac{4}{11} = 4 \div 11 = 0 \cdot 363636$ - - - - - - -

3. *A decimal number can be exact* ***or*** *inexact*

4. *An exact decimal or terminating decimal is a decimal that ends. This decimal is converted into a fraction as follows:*

(i) *Divide the decimal by one to get* $\frac{\text{decimal}}{1}$

(ii) *Multiply both top and bottom by* ***10*** *for every number after the decimal point*

5. *To add or subtract two decimal numbers, line up the decimal points and then workout.*

6. *To multiply or divide two decimal numbers, express the decimal numbers in fractions and then workout.*

EXAMPLES:

1. *Using a calculator, convert the following fractions into decimal numbers:*

(i) $\frac{1}{2}$ ***(ii)*** $\frac{1}{4}$ ***(iii)*** $\frac{3}{4}$ ***(iv)*** $\frac{7}{8}$ ***(v)*** $2\frac{5}{16}$ ***(vi)*** $3\frac{15}{64}$

2. *Using a calculator, convert the following fractions into recurring decimals:*

(i) $\frac{2}{9}$ ***(ii)*** $\frac{5}{11}$ ***(iii)*** $\frac{2}{15}$ ***(iv)*** $\frac{9}{22}$ ***(v)*** $2\frac{1}{6}$ ***(vi)*** $1\frac{7}{11}$

3. *Convert the following decimal numbers into fractions in their lowest terms:*

(i) 0·5 ***(ii) 0·25*** ***(iii) 0·75*** ***(iv) 0·375*** ***(v) 0·625***
(vi) 1·75 ***(vii) 2·35*** ***(viii) 0·3*** ***(ix) 0·37*** ***(x) 0·0225***

4. *Without using a calculator, evaluate:* $308 \cdot 6 + 20 \cdot 475 + 1 \cdot 36$

Soln:

$$\begin{array}{r} 308 \cdot 600 \\ 20 \cdot 475 \\ +\ 1 \cdot 360 \\ \hline 330 \cdot 435 \end{array}$$

5. *Without using a calculator, evaluate:* $13 \cdot 79 - 12 \cdot 547$

Soln:

$$\begin{array}{r} 13 \cdot 790 \\ -\ 12 \cdot 547 \\ \hline 1 \cdot 243 \end{array}$$

6. *Without using a calculator, evaluate:* $136 - 14 \cdot 54$

Soln:

$$\begin{array}{r} 136 \cdot 00 \\ -\ 14 \cdot 54 \\ \hline 121 \cdot 46 \end{array}$$

7. *Without using a calculator, evaluate:* $0 \cdot 5 \times 0 \cdot 08$

Soln:

$$0 \cdot 5 \times 0 \cdot 08 = \frac{5}{10} \times \frac{8}{100} = \frac{40}{1000} = 0 \cdot 04$$

8. *Without using a calculator, evaluate:* $0 \cdot 25 \times 0 \cdot 004$

Soln:

$$0 \cdot 25 \times 0 \cdot 004 = \frac{25}{100} \times \frac{4}{1000} = \frac{100}{100000} = 0 \cdot 001$$

9. *Without using a calculator, evaluate:* $\frac{0{\cdot}032}{0{\cdot}16}$

Soln:

$$\frac{0\cdot032}{0\cdot16} = \frac{32}{1000} \div \frac{16}{100} = \frac{32}{1000} \times \frac{100}{16} = \frac{2}{10} = 0\cdot2$$

10. *Without using a calculator, evaluate:* $\frac{0{\cdot}0125 \times 70}{0{\cdot}0035 \times 2{\cdot}5}$

Soln:

$$\begin{aligned}\frac{0{\cdot}0125 \times 70}{0{\cdot}0035 \times 2{\cdot}5} &= \frac{125}{10000} \times 70 \div \left(\frac{35}{10000} \times \frac{25}{10}\right) \\ &= \frac{125 \times 70 \times 10000 \times 10}{10000 \times 35 \times 25} \\ &= 5 \times 2 \times 10 \\ &= 100\end{aligned}$$

11. *Without using a calculator, evaluate:* $\frac{0{\cdot}15 \times 2{\cdot}8 \times 7{\cdot}5}{0{\cdot}007 \times 22{\cdot}5}$

Soln:

$$\begin{aligned}\frac{0{\cdot}15 \times 2{\cdot}8 \times 7{\cdot}5}{0{\cdot}007 \times 22{\cdot}5} &= \frac{15}{100} \times \frac{28}{10} \times \frac{75}{10} \div \left(\frac{7}{1000} \times \frac{225}{10}\right) \\ &= \frac{15 \times 28 \times 75 \times 100 \times 10}{100 \times 10 \times 10 \times 7 \times 225} \\ &= 4 \times 5 \\ &= 20\end{aligned}$$

12. *Without using a calculator, evaluate:* $\frac{(0{\cdot}2)^2 \times 0{\cdot}32}{(0{\cdot}4)^3}$

Soln:

$$\frac{(0{\cdot}2)^2 \times 0{\cdot}32}{(0{\cdot}4)^3} = \left(\frac{2}{10}\right)^2 \times \frac{32}{100} \div \left(\frac{4}{10}\right)^3$$

$$= \frac{4}{100} \times \frac{32}{100} \div \frac{64}{1000}$$

$$= \frac{4 \times 32 \times 1000}{100 \times 100 \times 64}$$

$$= \frac{2}{10}$$

$$= \mathbf{0 \cdot 2}$$

13. *Without using a calculator, find the square root of:*

(i) 0·09 (ii) 0·0081 (iii) 0·64 × 0·0121

Soln:

(i) $\sqrt{0 \cdot 09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = \mathbf{0 \cdot 3}$

(ii) $\sqrt{0 \cdot 0081} = \sqrt{\frac{81}{10000}} = \frac{9}{100} = \mathbf{0 \cdot 09}$

(iii) $\sqrt{0 \cdot 64 \times 0 \cdot 0121} = \sqrt{\frac{64}{100} \times \frac{121}{10000}} = \frac{8}{10} \times \frac{11}{100} = \frac{88}{1000} = \mathbf{0 \cdot 088}$

14. *Without using a calculator, find the cube root of:*

(i) 0·008 (ii) 0·027 × 0·125

Soln:

(i) $\sqrt[3]{0 \cdot 08} = \sqrt[3]{\frac{8}{1000}} = \sqrt[3]{\frac{2^3}{10^3}} = \frac{2}{10} = \mathbf{0 \cdot 2}$

(ii) $\sqrt[3]{0\cdot027\times0\cdot125} = \sqrt[3]{\frac{27}{1000}\times\frac{125}{1000}} = \sqrt[3]{\frac{3^3}{10^3}\times\frac{5^3}{10^3}} = \frac{3}{10}\times\frac{5}{10} = \frac{15}{100} = \mathbf{0\cdot15}$

RECURRING DECIMAL

Summary:

1. *A recurring decimal is a decimal with endless repeating digits after the decimal point.*

2. *A recurring decimal* ***0·272727 - - - - - - -*** *is the same as* $0\cdot\overline{27}$ *or* $0\cdot\dot{2}\dot{7}$

3. *A recurring decimal is converted into a fraction as follows:*

(i) *Form two equations in* ***x*** *with the same recurring part after the decimal point.*

(ii) *Eliminate the recurring part by subtracting those two equations*

EXAMPLES:

1. *Express* ***0·363636 - - - - - - -*** *as a fraction in its simplest form*

Soln:

$x = 0{\cdot}363636 - - - - - - -$

$100x = 36{\cdot}363636 - - - - - - -$

$$\begin{array}{rl} 100x = & 36\cdot363636 \text{ - - - -} \\ -\ x = & 0\cdot363636 \text{ - - - - -} \\ \hline 99x = & 36 \end{array}$$

$$x = \frac{36}{99}$$

$$\therefore \quad x = \frac{4}{11}$$

2. *Express* $0 \cdot \overline{891}$ *as a fraction in its simplest form*

Soln:

$x = 0{\cdot}891891\text{ - - - - - - -}$

$1000x = 891{\cdot}891891\text{ - - - - - - -}$

$$\begin{array}{l} 1000x = 891 \cdot 891891 \text{ - - - -} \\ -\ x = \quad 0 \cdot 891891 \text{ - - - - -} \\ \hline 999x = 891 \end{array}$$

$$x = \frac{891}{999}$$

$$\therefore \quad x = \frac{33}{37}$$

3. *Express* $1 \cdot \dot{2}\dot{7}$ *as a fraction in its simplest form*

Soln:

$x = 1{\cdot}272727\text{ - - - - - - -}$

$100x = 127{\cdot}272727\text{ - - - - - - -}$

$$\begin{array}{l} 100x = 127 \cdot 272727 \text{ - - - -} \\ -\ x = \quad 1 \cdot 272727 \text{ - - - - -} \\ \hline 99x = 126 \end{array}$$

$$x = \frac{126}{99}$$

$$\therefore \quad x = \frac{14}{11}$$

4. *Express* $0 \cdot \dot{1}\dot{6}$ *as a fraction in its simplest form*

Soln:

$x = 0{\cdot}16666\text{ - - - - - - -}$

$10x = 1{\cdot}6666\text{ - - - - - - -}$

$100x = 16{\cdot}6666\text{ - - - - - - -}$

$$\begin{array}{l} 100x = 16 \cdot 6666\text{ - - - -} \\ -\ 10x = 1 \cdot 6666\text{ - - - - -} \\ \hline 90x = 15 \end{array}$$

$$x = \frac{15}{90}$$

$$\therefore \quad x = \frac{1}{6}$$

5. *Express* ***0·0234234 - - - - - - -*** *as a fraction in its simplest form*

Soln:

$x = 0{\cdot}0234234\text{ - - - - - - -}$

$10x = 0{\cdot}234234\text{ - - - - - - -}$

$10000x = 234{\cdot}234234\text{ - - - - - - -}$

$$
\begin{array}{rcl}
10000x & = & 234 \cdot 234234\ \text{- - - -} \\
-\ 10x & = & 0 \cdot 234234\ \text{- - - - -} \\
\hline
9990x & = & 234
\end{array}
$$

$$x = \frac{234}{9990}$$

$$\therefore \quad x = \frac{13}{555}$$

6. *Express* $2{\cdot}014545\text{- - - - - - -}$ *as a fraction in its simplest form*

Soln:

$$x = 2{\cdot}014545\text{- - - - - - -}$$

$$100x = 201{\cdot}4545\text{- - - - - - -}$$

$$10000x = 20145{\cdot}4545\text{- - - - - - -}$$

$$
\begin{array}{rcl}
10000x & = & 20145 \cdot 4545\ \text{- - - -} \\
-\ 100x & = & 201 \cdot 4545\ \text{- - - - -} \\
\hline
9900x & = & 19944
\end{array}
$$

$$x = \frac{19944}{9900}$$

$$\therefore \quad x = \frac{554}{275}$$

7. *Find the exact value of* $2 \cdot 4\dot{1}\dot{2} - 0 \cdot 3\dot{2}$

Soln:

$$
\begin{array}{l}
\quad 2 \cdot 4141\ \text{- - - - - - - - - - -} \\
-\ \ 0 \cdot 3232\ \text{- - - - - - - - - -} \\
\hline
\quad 2 \cdot 0909\ \text{- - - - - - - - - -}
\end{array}
$$

$x = 2{\cdot}0909\ \text{- - - - - - -}$

$100x = 209{\cdot}0909\ \text{- - - - - - -}$

$$
\begin{array}{l}
100x = 209 \cdot 0909\ \text{- - - -} \\
-\ x = \quad 2 \cdot 09090\ \text{- - - - -} \\
\hline
99x = 207
\end{array}
$$

$\therefore \quad x = \frac{23}{11}$

EER:

1. *Express* ***0·242424 - - - - - - -*** *as a fraction in its simplest form*

2. *Find the exact value of* $1 \cdot \dot{8}\dot{2} - 0 \cdot \dot{2}\dot{8}$

3. *Without using a calculator, evaluate:* $\frac{0{\cdot}0035}{0{\cdot}07 \times 0{\cdot}2}$

4. *Express* $0 \cdot \dot{2}\dot{7}$ *as a fraction in its simplest form*

5. *Express* ***0·3181818 - - - - - - -*** *as a fraction in its simplest form*

6. *Without using a calculator, evaluate:* $\frac{0{\cdot}625 \times 0{\cdot}009}{0{\cdot}0045}$

7. *Express* $0 \cdot \overline{63}$ *as a fraction in its simplest form*

8. *Without using a calculator, find the square root of* ***0·0025 × 0·36***

9. *Without using a calculator, find the cube root of* ***0·216 × 0·125***

10. *Express* $0 \cdot 3\dot{7}$ *as a fraction in its simplest form*

11. *Express* $0 \cdot \overline{45}$ *as a fraction in its simplest form*

12. *Express* ***0·818181 - - - - - - -*** *as a fraction in its simplest form*

STANDARD FORM

Summary:

1. *Standard form is a way of expressing a number in the form* $A \times 10^{n}$ *where* $1 \leq A < 10$ *and* ***n*** *is an integer*

2. (i) *To express a number in standard form, we shift the decimal point until the digit part* ***A*** *is between* ***1*** *and* ***10.*** *This digit* ***A*** *has a decimal point placed after the first digit.*

(ii) *The power part* (10^{n}) *shows how many places to move the decimal point*

3. *The rules of indices apply to calculations in standard form*

EXAMPLES:

1. *Express the following numbers in standard form:*

(i) 25000 (ii) 3860 (iii) 568·3 (iv) 74·8 (v) 3·584 (vi) 4000

(vii) 0·435 (viii) 0·000263 (ix) 0·007 (x) 0·00356 (xi) 248×10^{3}

(xii) 4500×10^{-7} ***(xiii)*** $58 \cdot 4 \times 10^{-4}$ ***(xiv)*** $0 \cdot 027 \times 10^{-3}$

(xv) $0 \cdot 0062 \times 10^{-4}$ ***(xvi)*** $0 \cdot 0364 \times 10^{5}$

2. *By expressing each of the numbers in standard form, evaluate the following:*

(i) 0·0004 × 0·002 ***(ii) 0·005 × 0·00004*** ***(iii) 800000 × 0·0005***

(iv) $\frac{800}{0\cdot004}$ ***(v)*** $\frac{0\cdot00006}{2000}$ ***(vi)*** $\frac{0\cdot0009 \times 8000}{0\cdot002 \times 0\cdot3}$

3. *By expressing each of the numbers in the form* $a \times 10^n$ *where* ***n*** *is even, evaluate the following:*

(i) $\frac{0\cdot81}{0\cdot0027}$ ***(ii) 0·02 × 0·0015*** ***(iii) 1500 × 40000***

4. *By expressing each of the numbers in the form* $a \times 10^n$ *where* ***n*** *is even, find the square root of:*

(i) 0·25 × 0·64 ***(ii) 0·16 × 0·09*** ***(iii) 0·0036 × 0·25*** ***(iv) 0·0081 × 0·0004***

5. *Without using a calculator, evaluate the following give your answer in standard form:*

(i) $(2 \times 10^2) \times (6 \times 10^5)$ ***(ii)*** $(4 \times 10^{-3}) \times (5 \times 10^4)$

(iii) $(9 \times 10^{-2}) \times (4 \times 10^{-3})$ ***(iv)*** $(15 \times 10^{-3}) \times (20 \times 10^7)$

6. *Without using a calculator, evaluate:* $\frac{1\cdot21 \times 10^{-2} \times 40}{2\cdot2 \times 11}$

7. *Without using a calculator, evaluate:* $\frac{2{\cdot}25 \times 10^{-3} \times 3{\cdot}9}{1{\cdot}3 \times 1{\cdot}5}$

EER:

1. *Without using a calculator, evaluate:* $\frac{0{\cdot}0035}{0{\cdot}07 \times 0{\cdot}2}$

2. *Without using a calculator, evaluate:* $\frac{0{\cdot}625 \times 0{\cdot}009}{0{\cdot}0045}$

3. *Without using a calculator, evaluate:* $\frac{1{\cdot}69 \times 10^{-2} \times 1{\cdot}25}{1{\cdot}3 \times 2{\cdot}5}$

4. *Without using a calculator, evaluate:* $\frac{5600}{80000}$ *give your answer in the form* $a \times 10^{n}$ *where* $1 \leq a < 10$ *and* n *is an integer*

APPROXIMATIONS AND ESTIMATIONS

This deals with rounding off a number correct to a given number of decimal places ***or*** *significant figures.*

Summary:

1. *Decimal places are counted from the decimal point. Thus* ***74·564*** *has* ***3*** *decimal places*

2. *To round up a number to a given number of decimal places, the digit in the next decimal place must be greater* ***or*** *equal to* ***5.*** *Thus to* ***two*** *decimal places* ***8·6287 ≈ 8·63*** *and* ***3·3847 ≈ 3·38.***

3. (i) *Any digit that is not zero is significant.*

(ii) *Zeros at the beginning and at the end of a number are not significant, well as the zeros between non–zero digits are significant.*

(iii) *Significant figures are counted from the first non-zero digit. Thus to* ***three*** *significant figures* ***8·6387 ≈ 8·64, 3·06847 ≈ 3·07, 0·060287 ≈ 0·0603*** *and* ***4028400 ≈ 4030000***

EXAMPLES:

1. *Round off the following numbers to* ***2*** *decimal places.*

(i) *49·376* ***≈ 49·38*** *(to 2 d.p)*

(ii) *8·264* ***≈ 8·26*** *(to 2 d.p)*

(iii) *5·997* ***≈ 6·00*** *(to 2 d.p)*

NB : *Writing zeros in* ***(iii)*** *above shows that the number is written to* ***2 d·p.***

2. *Round off the following numbers to* ***3*** *significant figures.*

(i) *8·0046* ***≈ 8·00*** *(to 3 s.f)* ***3045***

(ii) *0·07058* ***≈ 0·0706*** *(to 3 s.f)*

(iii) *4526* ***≈ 4530*** *(to 3 s.f)*

(iv) *3048* ***≈ 3050*** *(to 3 s.f)*

(v) *24170* ***≈ 24200*** *(to 3 s.f)*

3. *Using a calculator, convert the fraction* $\frac{5}{7}$ *into a decimal number correct* ***4*** *decimal places*

4. *Using a calculator, find the square root of* ***21*** *correct to* ***3*** *decimal places*

5. *Using a calculator, find the cube root of* ***4*** *correct to* ***3*** *significant figures*

6. *Given that* $\pi = 3{\cdot}142$ *and* $r = 3.3$, *find the value of* $A = \pi r^2$ *correct to* ***4*** *significant figures*

7. *Given that* $\pi = 3{\cdot}14$, $r = 2{\cdot}4$ *and* $h = 3$, *find the value of* $V = \pi r^2 h$ *correct to* ***2*** *decimal places*

8. *Given that* $\pi = 3{\cdot}14$ *and* $r = 1.6$, *find the value of* $V = \frac{4}{3}\pi r^3$ *correct to* ***4*** *significant figures*

ERR:

1. *Round off the value of* ***0·070368*** *to:*

(i) 3 *decimal places.*

(ii) 4 *significant figures.*

2. (i) *Round off* ***2·09974*** *to* ***3*** *d·p.*

(ii) *Round off* ***0·070865*** *to* ***3*** *s·f.*

__(iii)__ Round off __5·0036__ to __3__ s·f.

__(iv)__ Round off __364800__ to __3__ s·f.

__(v)__ Round off __3·9963__ to __2__ d·p

__(vi)__ Round off __0·4586__ to __2__ d·p

__(vii)__ Round off __452700__ to __3__ s·f

__2.__ A square has an area of __20__ cm^2__.__ Find the length of its side, correct to __1d.p.__

__4.__

__5.__

__6.__

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